Cooperative games in building resilience for flooding river

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Game Theory

- ***Whenever human beings/needs match or conflict, a game is being played***

- ***We have games on***

- **driving : drivers in heavy traffic**
- **bargaining: negotiating of wage**
- **politics: opposing candidates in election**
- **economics: grocer deciding corn price**
- **auctioning: the bidding for a forest cut**
- ***building resilience: fronting a possible disaster***

game against natural hazards

Game

A game is a

- formal mathematical model describing
- the strategic interaction among
- several agents
- and implying that
- the result obtained by an agent
- depends on
- his/her actions and on
- the other agents' actions

Game

needs:

- **Two or more players,**
- **Rules,**
- **Payoff vector** **which represents the utility that each player can obtain with respect to all possible combination of strategies**

Cooperative or not, coalitive or not games

- **Non cooperative games: single participants make their choices on the base of own individual reasonings**
- **Cooperative games: all participants, as players, aggragate in order to realize an advantage (on fronting better a natural hazard)**
- **Coalitive games: part of participants, as players, aggregate to play against the rest of the players in order to realize higher utility (power)**
- **Non coalitive games: participants are antagonistic players**

Cooperative games

- In cooperative games it is possible to form coalitions, on which each participant anyway receives a better payoff (more resilient situation). In the case of all participants we speak of big coalition.

- How to find the right allocation of the gain (saved resources) among the partecipants? Each acceptable allocation is called **imputation**.

- On building resilience we can evaluate, by Shapley value, the imputation/marginal contribution to the coalition of each partecipant/town, which it is depending only from the power he can obtain to stay alone or with somebody else.

Shapley value ϕ of imputation

ϕ is a rule assigning to the n-player game *v*, an *n*-dimensional vector $\phi(v)=[\phi_1(v), \phi_2(v),\ldots,\phi_n(v)]$ with the properties:

Efficiency: $\sum_{i=1}^{n} \phi_i(v) = v(N)$, the total win is interely divided among coalition members

Simmetry: the imputation is not depending from the name of the player, but from its position and role in the play

Linearity, if *u*,*v* are two games and α, β two scalars then:
$\phi(\alpha u+\beta v) = \alpha\, \phi(u) + \beta\, \phi(v)$

Irrilevance of i-th dummy player (null contribute) then $\phi_i(v)=0$.

Shapley index

If C is coalition with c players, there is the theorem:

***It exists an unic function satisfying the previuos four axioms, and its expression is*:**

$$\phi_i(v) = \sum_{C \subseteq N} \frac{(c-1)!(n-c)!}{n!} \; [v(C)\text{-}v(C\text{-}\{i\})]$$

The value $\mathbf{[v(C)\text{-}v(C\text{-}\{i\})]}$ is the marginal contribute of i-th player to C coalition.

The Shapley index is suitable in many real situations, expecially when we want compute the marginal contribution of each partecipant/town on building resilience.

Imputation in building resilience coalition

A flooding river caused damages in four towns for U=23.000 as U_1=5.000, U_2=4.000, U_3=6.000, U_4=8.000. A project, in order to make resilient that towns, costs T=10.000 and has the four towns as possible participants, each providing resources exactly the money saved as taxation/costs of damages from the previous flooding. Each resource is not enough for covering the complete project, realisable only in some coalitions. Certainly the costs and advantages must be proportioned to the endured damages.

The marginal contribution on the possible coalitions are:

	U_1	U_2	U_3	U_4
U_1	5	9	11	13
U_2	9	4	10	12
U_3	11	10	6	14
U_4	13	12	14	8

$v(1) = v(2) = v(3) = v(4) = 0$

$v(1,2) = v(2,3) = 0$, $v(1,3) = 1000$,
$v(3,4) = 4000$, $v(2,4) = 2000$, $v(1,4) = 3000$,

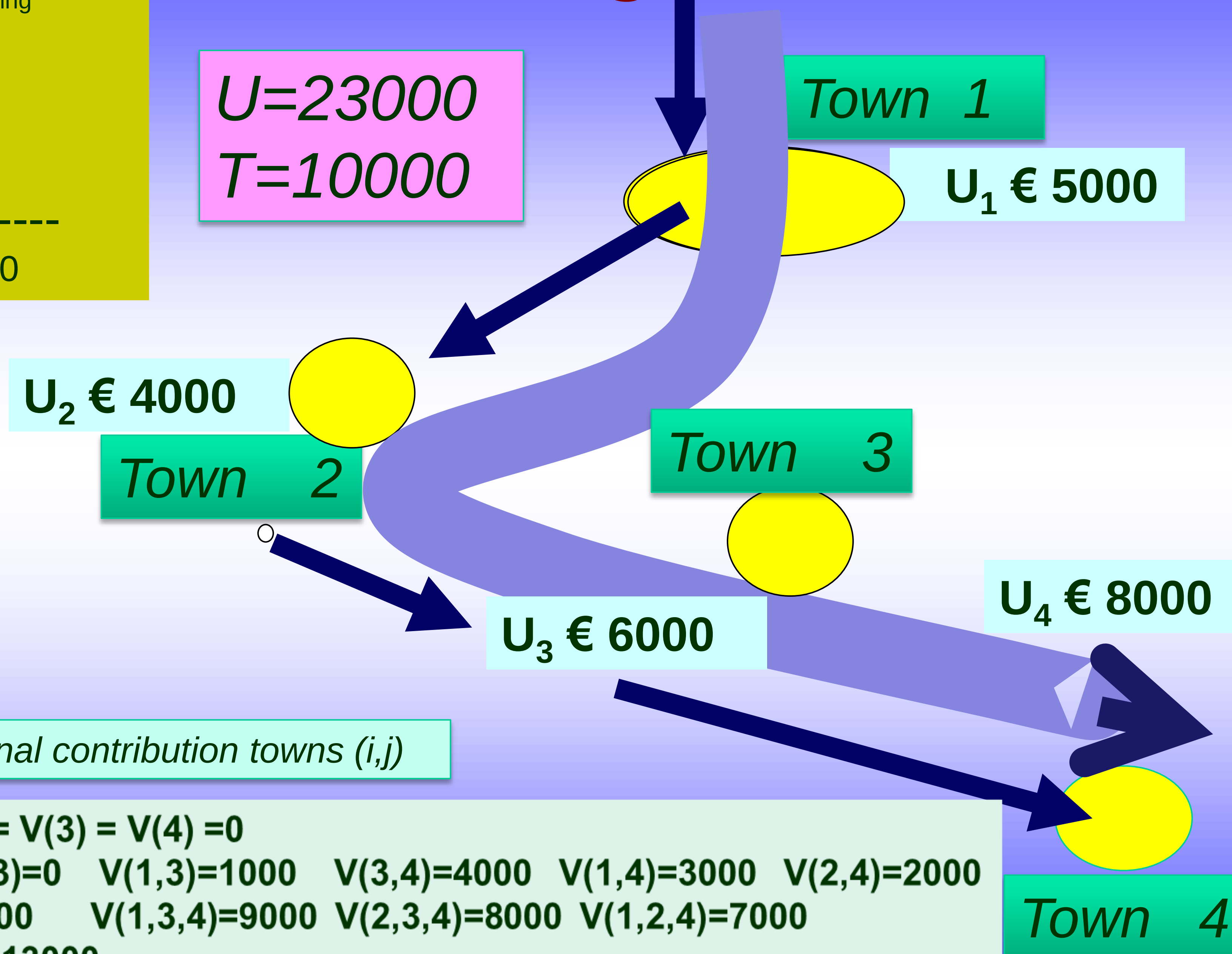
$v(1,2,3) = 5000$, $v(1,3,4) = 9000$,
$v(2,3,4) = 8000$, $v(1,2,4) = 7000$,
$v(\mathbf{1,2,3,4}) = 13000$

Flooding river

Town | Damages U
|| previous flooding

1 : € 5000
2 : € 4000
3 : € 6000
4 : € 8000

Total € 23000



V(i,j) =Marginal contribution towns (i,j)

V(1) = V(2) = V(3) = V(4) =0
V(1,2)=V(2,3)=0   V(1,3)=1000   V(3,4)=4000   V(1,4)=3000   V(2,4)=2000
V(1,2,3)=5000   V(1,3,4)=9000   V(2,3,4)=8000   V(1,2,4)=7000
V(1,2,3,4) = 13000

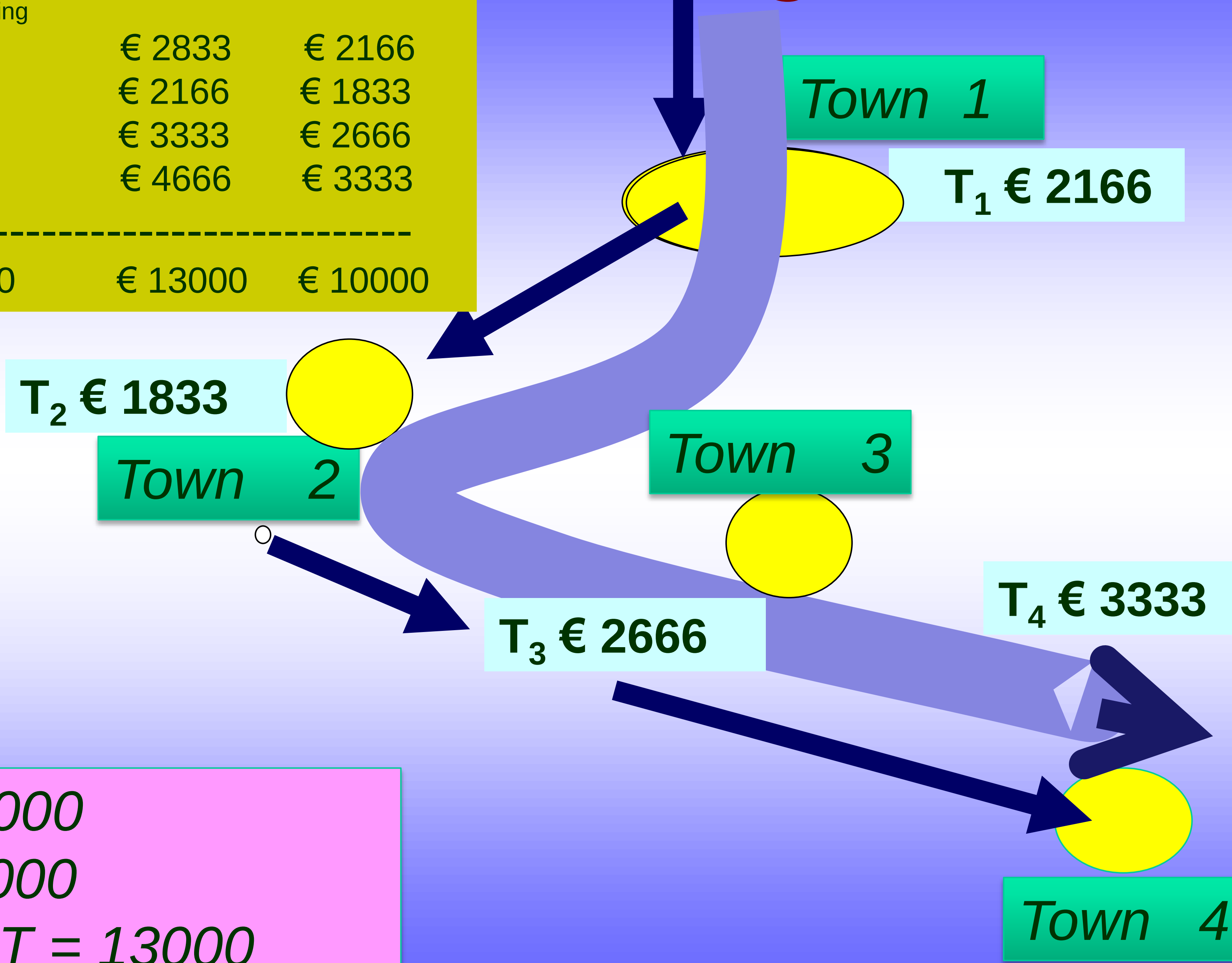
		Contribution of	the i-th town to the	coalitions
Entering town	c=1	c=2	c=3	c=4
1	v(1)-v(0)=0 totale 0	v(1,2)-v(2)=0 v(1,3)-v(3)=1 v(1,4)-v(4)=3 totale 4	v(1,2,3)-v(2,3)=5 v(1,2,4)-v(2,4)=5 v(1,3,4)-v(3,4)=5 totale 15	v(1,2,3,4)- v(2,3,4)=5
2	v(2)-v(0)=0 totale 0	v(1,2)-v(1)=0 v(2,3)-v(3)=0 v(2,4)-v(4)=2 totale 2	v(1,2,3)-v(1,3)=4 v(1,2,4)-v(1,4)=4 v(2,3,4)-v(3,4)=4 totale 16	v(1,2,3,4)- v(1,2,4)=4
3	v(3)-v(0)=0 totale 0	v(1,3)-v(1)=1 v(2,3)-v(2)=0 v(3,4)-v(4)=4 totale 5	v(1,2,3)-v(1,2)=5 v(2,3,4)-v(2,4)=6 v(1,3,4)-v(1,4)=6 totale 17	v(1,2,3,4)- v(2,1,4)=6
4	v(4)-v(0)=0 totale 0	v(1,4)-v(1)=3 v(2,4)-v(2)=2 v(3,4)-v(3)=4 totale 9	v(1,2,4)-v(1,2)=7 v(1,3,4)-v(1,3)=8 v(2,3,4)-v(2,3)=8 totale 23	v(1,2,3,4)- v(2,1,3)=8
Coeff.	$\frac{0!\,(4-1)!}{4!}=\frac{1}{4}$	$\frac{1!\,2!}{4!}=\frac{1}{12}$	$\frac{2!\,1!}{3!}=\frac{1}{12}$	$\frac{3!\,0!}{4!}=\frac{1}{4}$

- The Shapley values on sharing the savings of ☞ =13000 are:
- ϕ_1=1/12 (1000+3000) + 1/12 (5000+5000+5000) + 1/4 (5000) = 2833
- ϕ_2=1/12 (2000) + 1/12 (4000+4000+4000) + 1/4 (4000) = 2166
- ϕ_3=1/12 (1000+4000) + 1/12 (5000+6000+6000) + 1/4 (6000) = 3333
- ϕ_4=1/12 (3000+2000+4000)+1/12 (7000+8000+8000)+1/4 8000=4666
-
- The shared/proportioned financial support $T = U - \phi$ becomes
- T_1 = 5000-2833 = 2166
- T_2 = 4000-2166 = 1833
- T_3 = 6000-3333 = 2666
- T_4 = 8000-4666 = 3333.

- Savings/taxation $\boldsymbol{\phi = U\text{-}T}$ are equal to the difference between damages and financial support and it is shared among participant proportionally to their contributions to the coalition.

Flooding river

Town	U=damages || previous flooding	☞=gain Shapley value	T=finSup
1 :	€ 5000	€ 2833	€ 2166
2 :	€ 4000	€ 2166	€ 1833
3 :	€ 6000	€ 3333	€ 2666
4 :	€ 8000	€ 4666	€ 3333
Total	€ 23000	€ 13000	€ 10000



U = 23000
T = 10000
☞ = U-T = 13000

Coalitive model on building resilience

i,j=1,2,…N towns or communalities in coalition or not
h =1,2,…H public sectors/braches in coalition or not (schools, civil infrastructures, commercial, industrial, touristic activities, health care system, real estate), spreaded in each town.

T = total amount as governamental financial support in order to correct resilience and given after a disaster

T_i = governamental financial support assigned to the *i* -th town

$$\sum_i c_i T_i = T \qquad \sum_h w_{ih} T_{ih} = T_i$$

w_{ih} weight of the *h*-th sector in the *i*-th town computed in MCDA by the use of Electre III credibility index

c_i weight/virtuosity of *i*-th town computed in MCDA by the use of Electre III credibility index

Flooding river resilience model

U = total amount of damages from last disaster is more than the financed amount *T*

☞ = *U – T is the possible saving/gaining amount or the sum to be financed by local taxation.*

saving = damages – financial support

to be shared among all the town in coalition by Shapley value

U_i = damages in the *i*-th town

U_{ih} = damages in the *h*-th sector in the *i*-th town

$$U_i = \sum_h w_{ih} U_{ih} \qquad U = \sum_i \sum_h w_{ih} U_{ih}$$

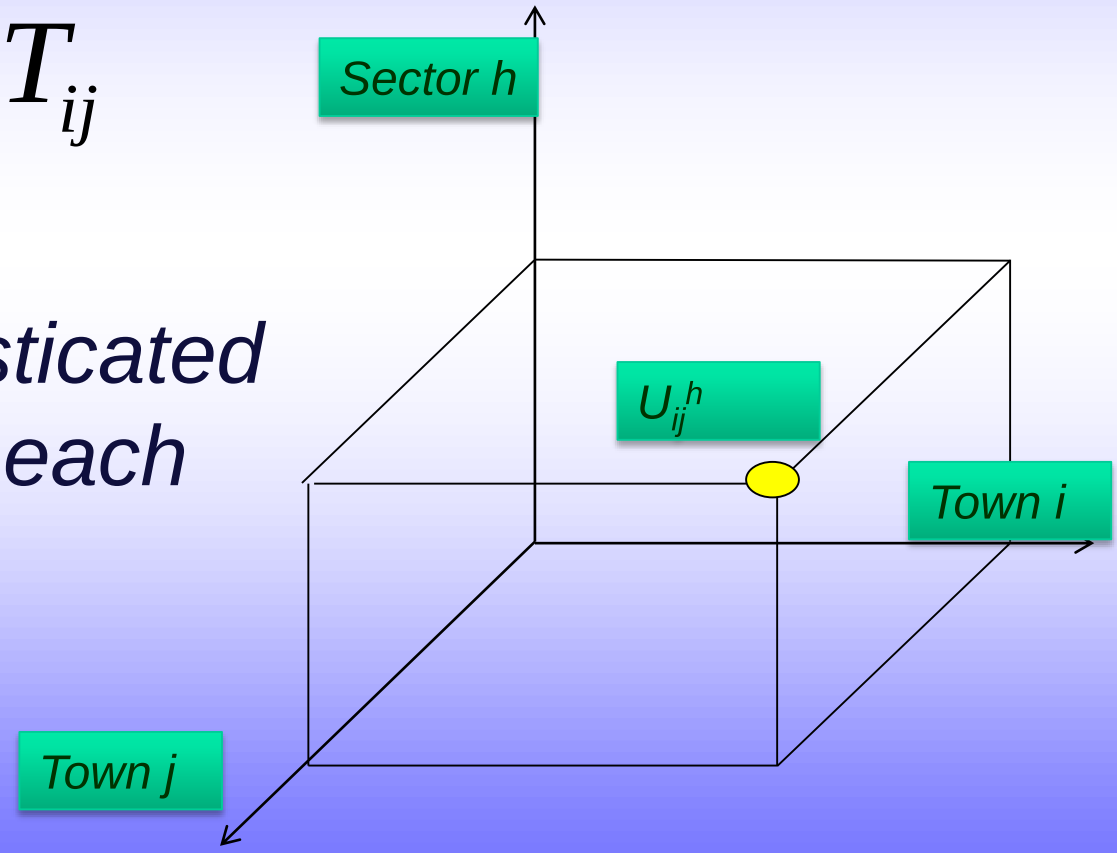
$$\Phi = U - T = \sum_i \sum_h w_{ih} U_{ih} - \sum_i c_i T_i$$

Imputation for each couple of towns

Gain/saving must be splitted among each possible couple of towns in coalition

$$\Phi_{i,j}(v) = U_{ij} - T_{ij}$$

and, in a more sofisticated model, relatively to each couple of towns for each sector



... and relating to each sector

Damages U_{ij}^{h} on the *h*-th sector combined between coupled (*i*,*j*) towns

j-th sector	U_{1j}	U_{2j}	U_{3j}	U_{4j}
U_{1j}	5	9	11	13
U_{2j}	9	4	10	12
U_{3j}	11	10	6	14
U_{4j}	13	12	14	8

in yellow the couple over 10

Financial support T_{ij}^{h} on the *h*-th sector combined on coupled (*i*,*j*) towns

$$\Phi_{ij}^{h}(v) = U_{ij}^{h} - T_{ij}^{h}$$

j-th sector	T_{1j}	T_{2j}	T_{3j}	T_{4j}
T_{1j}	2	4	7	5
T_{2j}	1	3	5	6
T_{3j}	4	7	1	8
T_{4j}	7	5	6	2

Conclusion

Game theory is a powerful tool in order to describe strategic interactions, necessary to guarantee the respect of some specific principles and to promote the partecipation to the agreements on building resilience.

In order to promote cooperation on building resilience framework, motivation of tutela and suitable incentives are strongly combined

Incentives, given in order to promote agreements, are addressed to forecast a more resilient situtation and to substene the abatiment of criticalities in territorial resilience.